CORRELATION BETWEEN LIQUID LIMIT AND SHRINKAGE LIMIT

Gerald Maregesi

Email: gerald.maregesi@aesl.co.tz

ABSTRACT

Detailed analysis of 181 Atterberg limits test results collected from literature demonstrates that shrinkage limit is the plasticity characteristic of soil. The main factor affecting the shrinkage limit is the specific surface area of the clay minerals. As the specific surface area increases, the shrinkage limit decreases. Two models for estimating the shrinkage limit have been developed. The first model is based on the average slope of the fall cone flow curve (20/LL), while the second model uses the liquid limit to estimate the shrinkage limit of the soil. In line with AASHTO T92-88, the reproducibility of the shrinkage limit test results is ±6.8. 95% of the shrinkage limit results computed using the proposed models plotted within the statistical testing bound, indicating the robustness of the proposed model and its potential for estimating the shrinkage limit of the soil.

INTRODUCTION

The shrinkage limit, one of the Atterberg limits parameters, is typically the lowest, with the liquid limit being the highest. The shrinkage limit has three distinct definitions. Firstly, it is the maximum water content at which any further reduction in water content will not cause a decrease in the volume of the soil mass. Secondly, it is the moisture content at which the soil transitions from a semi-solid to a solid state. Lastly, it is the minimum moisture content at which the soil can still be fully saturated. The mercury method, outlined in BS 1377: Part 2:1990, AASHTO T92-88, and ASTM D427-98, are the most commonly used methods for determining the shrinkage limit. However, it's important to note that mercury is expensive and hazardous, posing significant risks. An alternative method, ASTM D4943-02, which uses wax, has been introduced. Both methods are operator-dependent and time-consuming. As a result, Casagrande's method, which estimates the shrinkage limit based on the U-line and A-line, is often used to calculate the shrinkage limit of the soil. This method has been reported to be close to the determined values of the shrinkage limit [Holtz and Kovacs (1981)]. It is worth noting that this method of estimating the shrinkage limit utilizes the plasticity index and liquid limit, suggesting a correlation between the shrinkage limit, plastic index, and liquid limit.

Several alternative methods have been proposed to determine the shrinkage limit, such as sand replacement, wax method, and reverse extrusion, all aimed at improving the accuracy of the mercury method [Cerato (2006), Prakash and Sridharan (2011), Prakash and Sridharan (2012), Kayabali (2015)]. Maregesi (2022) proposed equations that utilize the British fall cone slope to estimate the shrinkage limit.

This paper improves the method of estimating the shrinkage limit proposed by Maregesi(2022), whereby the fall cone slope used in the previously proposed model was computed using Equation 1. This paper introduces the average fall cone slope, defined in Equation 2 (Maregesi, 2023), as a more reliable parameter for estimating the soil's shrinkage limit. The fall cone slope varies as moisture content and penetration values decrease, which can introduce errors when using Equation 1 (Maregesi, 2022). However, the average fall cone slope computed using equation 2 is invariant, offering a more robust approach and thus improving the model's accuracy when estimating the soil's shrinkage limit.

$$S = \frac{25 - 5}{w_{25} - w_5} = \frac{20}{w_{25} - w_5} \dots\dots (1)$$

$$S = \frac{20}{LL} \dots\dots (2)$$

Where,

'W_{25}' and 'W_5' are water contents corresponding to 25 mm and 5 mm penetration values. 'LL' is the liquid limit, and 'S' is the slope of the fall cone flow curve.

DEVELOPMENT OF A MODEL FOR COMPUTING SHRINKAGE LIMIT USING AVERAGE FALL CONE SLOPE

Maregesi (2022) proposed an equation for estimating the shrinkage limit. This equation is based on the British fall cone slope, a parameter known to vary with moisture and penetration values (Maregesi, 2022). It's worth noting that the slope was computed using a penetration value of 5 mm and its corresponding moisture content, as shown in Equation 1. However, the fall cone slope changes as the penetration values change, suggesting that a different fall cone slope could have been obtained if the penetration values of 3 mm and its corresponding moisture content were used instead of 5 mm. Based on the hypothesis that the moisture content of zero corresponds to a penetration value of 0 mm, the fall cone slope computed using coordinates <0,0> and <LL,20> defines the invariant average fall cone slope, which, in turn, represents the average rate of change of shear strength of the soil as moisture content varies (Maregesi, 2023). The average fall cone slope, a key parameter in this study, is computed using Equation 2.

For developing a model of computing the shrinkage limit, 181 Atterberg limits data were collected from literature and analyzed [Prakash & Sridharan (2011), Prakash & Sridharan (2012), Kayabali (2012), Kayabali & Yaldiz(2012), Prakash *et al.* (2015) and Vincent *et al.* (2021).

Casagrande cup and fall cone methods of determining the liquid limits give different values of liquid limit; thus, the computed plasticity index also differs. However, in this study, it has been assumed that both methods yield the same results.

The Atterberg limits, including liquid, plastic, and shrinkage limits, are the most common. These limits are not straightforward but arbitrary boundary values, marking the soil's transition from one state to another. Statistically, establishing a reliable correlation between these boundary values and other parameters is a complex task or impossible. However, this challenge can be overcome using a range of moisture content, such as plasticity and shrinkage indexes. For instance, most geotechnical correlations are based on the moisture content range between the boundary values, such as the plasticity index, which is the arithmetic difference between the liquid and plastic limits and is correlated to several soil parameters, such as expansiveness potential.
Similarly, the shrinkage index, the arithmetic difference between the plastic and shrinkage limits, can be used to correlate soil shrinkage with other geotechnical properties. Another parameter, the arithmetic difference between the liquid and shrinkage limits (LL-SL), is rarely used and was not defined by Atterberg limits. However, it can be defined as the moisture content range in which the soil transitions from a liquid state to a solid state. For this study, this arithmetic difference (LL-SL) is termed the solidity index (SD_i).

Maregesi (2022) showed an excellent correlation between the fall cone slope and the solidity index (LL-SL) (R^2=0.99). The slope used was computed using Equation 1. The correlation between the average fall cone slope, as defined in Equation 2, and the solidity index was carried out during this study. As expected, the solidity index is highly correlated with the average fall cone slope, as evidenced by the achieved coefficient of determination (R^2) of 0.9932 when the data is fitted using the rational function shown in equation 3. This correlation between the solidity index and the average fall cone slope is shown in Figure 1. Based on this correlation, the average fall cone slope can be used to compute the shrinkage limit using Equation 4.

Maregesi (2023) showed that the clay minerals can be delineated using the average fall cone slope. The soil with an average fall cone slope of less than 0.16 is montmorillonite, while mixed clay minerals and illite clay have an average fall cone slope within the range of 0.16-0.56, while the average fall cone slope of more than 0.56 is the kaolinite clay. Based on the definition of the average fall cone slope, the liquid limit can also be used as a simple method of delineating the clay minerals. The soil with a liquid limit of more than 125 is montmorillonite, while the mixed clay minerals, including illite clay, have a liquid limit of 36-125. In contrast, the kaolinite has a maximum liquid limit of 36.
From Figure 1, it can be seen that montmorillonite has a minimum solidity index of 108. In contrast, the mixed clay mineral, including illite clay, has a solidity index of 14 to 108. The kaolinite has a solidity index of less than 14. This suggests that the shrinkage limit

depends on the mineralogical composition of the clay minerals. Furthermore, since different clay minerals have distinct values of specific surface area, it is evident that the shrinkage limits depend on the surface area of the clay minerals. As the surface area of the clay minerals increases, the solidity index also increases (see Figure 1)

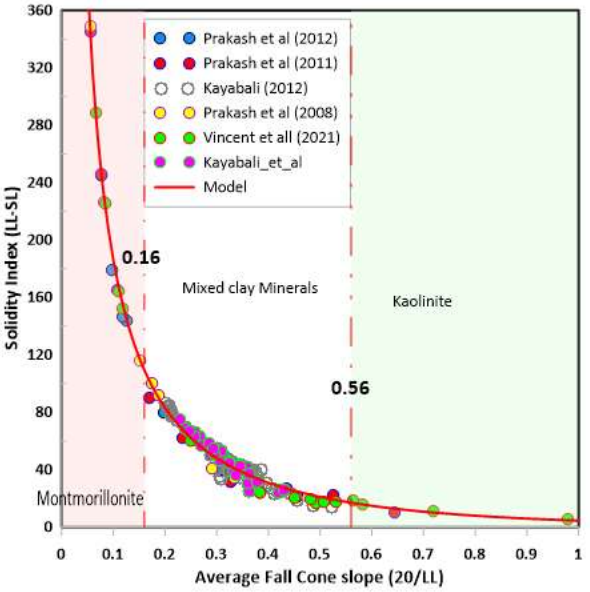

Figure 1: Correlation between the average fall cone slope and solidity index (R^2=0.9932)

$$LL - SL = \frac{288.689 - 295.0124(S^{0.1})}{1 - 4.2158S^{0.1} + 4.1937S^{0.2}} \dots (3)$$

Making the Shrinkage limit (SL) the subject, 'SL' can be computed using equation 4.

$$SL = LL - \left(\frac{288.689 - 295.0124(S^{0.1})}{1 - 4.2158S^{0.1} + 4.1937S^{0.2}}\right) \dots (4)$$

Where:

'SL' is the shrinkage limit, and 'S' is the average fall cone slope, as defined in Equation 2.

COMPUTATION OF SHRINKAGE LIMIT USING THE PROPOSED MODEL

The shrinkage limit was computed using Equation 4. The model's accuracy was assessed using a residual plot, a method widely used to identify any issues with regression and a crucial metric in assessing the model's accuracy. Figure 2 provides a detailed illustration of the computed shrinkage limit and the residual, demonstrating that 95% (172 out 181) of the calculated shrinkage limit test results are within the statistical testing bound of ±6.8 specified in AASHTO T 92-88. The residual values are evenly and randomly spaced around the horizontal axis.

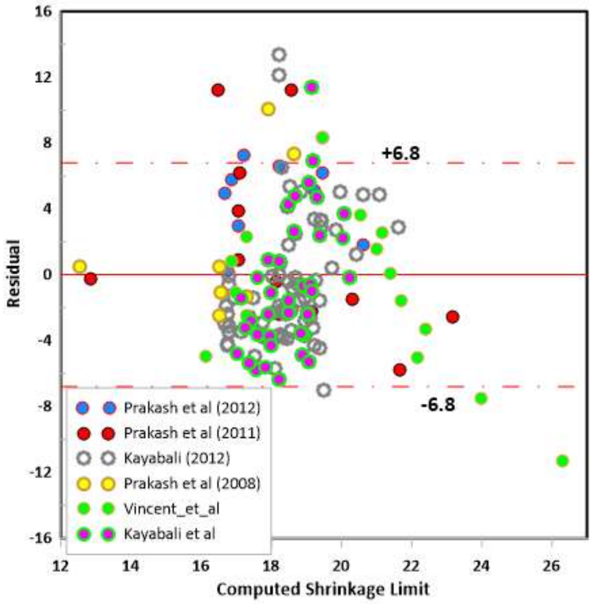

Figure 2: comparison between the computed shrinkage limit and the residual

Figure 3 compares the calculated and determined shrinkage limit, revealing that the results are randomly scattered on both sides of the line of equality, suggesting that the model fits the data reasonably well. Figure 4 presents a residual histogram, which indicates that 95% of the residuals are within the statistical testing bound of ±6.8 and are symmetrical about the origin, satisfying the assumption of residual normality. The residual has a high density of points close to the origin and a low density of points away from the origin, confirming that the residual is independent and normally distributed. Therefore, based on the test results analyzed and presented in Figures 1-4 suggest that the shrinkage limit values obtained from the mercury displacement method and the one computed using equation 4 are within acceptable statistical testing bounds, suggesting that the proposed model can estimate the shrinkage limit with reasonable accuracy and can be used in practice to estimate the shrinkage limit of the soil.

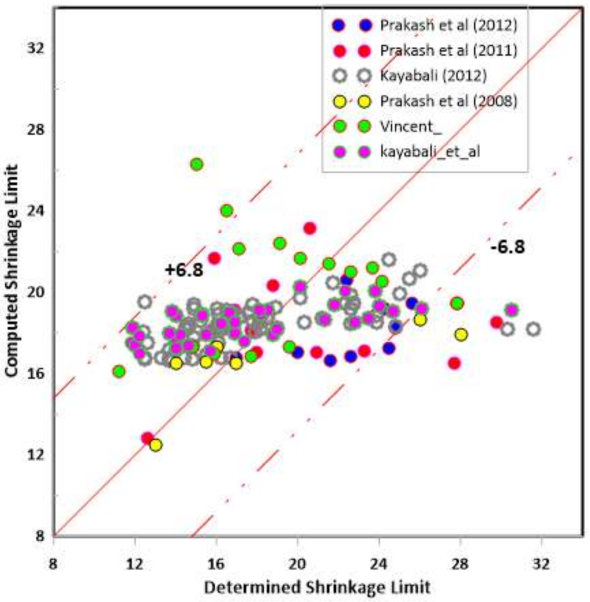

Figure 3: Comparison between determined and computed shrinkage limit

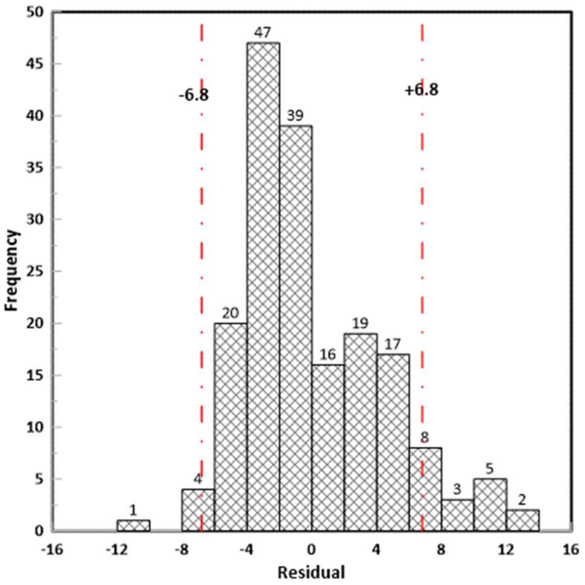

Figure 4: Residual histogram

CORRELATION BETWEEN LIQUID LIMIT AND SHRINKAGE LIMIT

Equation 4 shows that the shrinkage limit can be computed using two parameters: the liquid limit and the average fall cone slope, which is also a function of the liquid limit as defined in Equation 2. Therefore, by replacing the average fall cone slope (S) in Equation 4 with Equation 2, the shrinkage limit can be estimated using the liquid limit as the sole predictor. The resulting expression is shown as equation 5.

$$SL = LL - \left(\frac{288.689 - 398.055\left(\frac{1}{LL}\right)^{0.1}}{1 - 5.6883\left(\frac{1}{LL}\right)^{0.1} + 7.6349\left(\frac{1}{LL}\right)^{0.2}} \right) \ldots (5)$$

Figure 5 shows the variation between the liquid limit and shrinkage limit, from which it can be seen that there is no correlation between the liquid limit and shrinkage limit. Although there is no direct correlation between the liquid limit and shrinkage limit when the shrinkage limit is computed using equation 5, it can be seen that 95% of the test results are plotting within the statistical testing bound of ±6.8 as given in AASHTO T92-88 suggesting that the liquid limit can be used to estimate the shrinkage limit of the soil with reasonable accuracy and within the statistical testing bounds. The lack of correlation between liquid limit and shrinkage limit is not a surprise since liquid limit, plastic limit, and shrinkage limit are boundary values that provide no useful information on the plasticity or shrinkability properties of the geomaterial. The plasticity properties of the soil can be provided by the range of the moisture content from which the soil changes from one state to another. The plasticity properties of the soil can be explained by the plasticity index, which is the arithmetic difference between the liquid and plastic limits. The plastic limit, which is the boundary value, cannot be used to describe the plasticity properties of the soil.

Similarly, the shrinkage characteristic of the soil can be explained using a range of moisture content, such as the solidity index (LL-SL) or shrinkage index, defined as the arithmetic difference between the plastic limit and the shrinkage index (PL-SL). Therefore, the shrinkage limit is a boundary value that cannot be used to describe the shrinkage behavior of the geomaterial. Because the solidity index is highly correlated to the average fall cone slope (20/LL), this correlation suggests that the shrinkage limit is the plasticity properties of the soil. The average fall cone slope (20/LL) is correlated to the solidity index, and for most organic soil, the average fall cone slope is correlated to the plasticity index; impliedly, the shrinkage limit is also correlated to the plasticity index. Furthermore, since the plasticity index is imparted to the soil by active clay content, the shrinkage limit is also correlated to the active clay content of the soil.

Figure 5 shows that based on the liquid limit, it is possible to estimate the shrinkage limit of different clay minerals, namely kaolinite, mixed clay minerals, and illite and montmorillonite. It can be deduced that kaolinite clay minerals have a shrinkage limit of more than 21.5±6.8, while the mixed clay minerals and illite have a shrinkage limit within the range of 16.7±6.8-21.5±6.8 and the montmorillonite clay has shrinkage limit of less than 16.7±6.8.

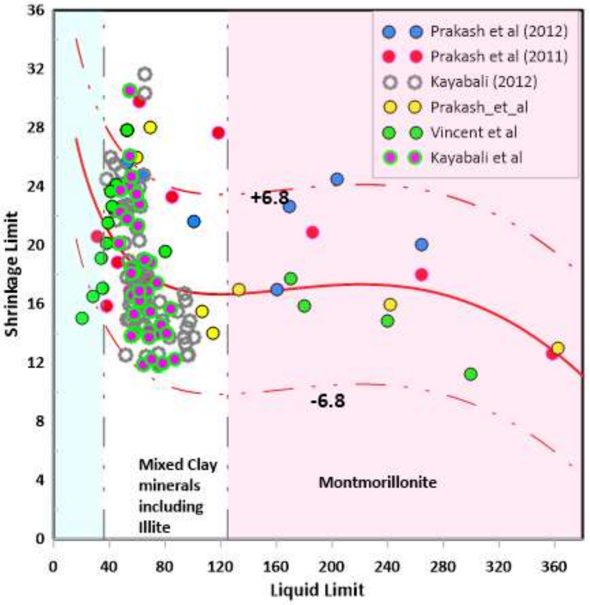

Figure 5:variation of liquid limit and shrinkage limit

VALIDATION OF THE MODEL

Sridharan et al. (2000), using clay-clay, sand-clay, and non-cohesive soil mixtures, presented a variation of bentonite content and shrinkage limit or liquid limit. The second series of test results presented a variation of the blend of red earth soil of low plasticity with brown soil of high plasticity. As expected, the liquid limit monotonically increased as the bentonite/cohesive brown soil content increased. However, the shrinkage limit decreased as the bentonite content increased and later increased. The same trend was observed for the brown soil and red earth blend. Based on this trend, it was concluded that the shrinkage limit is not a plasticity characteristic of the soil; if it could have been a characteristic property of soil, the shrinkage limit of the soil mixture was expected to decrease as the liquid limit increased.

Figure 5 shows a variation between the liquid and shrinkage limits developed from Equation 5. It can be seen that the correlation between the liquid limit and shrinkage limit is complex and does not decrease monotonically as the liquid limit increases. The rate of change of shrinkage limit with a liquid limit can be explained better by differentiating equation 5 or numerically differentiating the best-fit curve shown in Figure 5. Figure 6 shows the rate of change of shrinkage limit with liquid limit developed after numerically differentiating the best-fit curve given in Figure 5. It can be seen that there are three distinct zones. Zone 1 covers a liquid limit of less than 123, suggesting that this zone covers kaolinite clay and mixed clay minerals, including illite. In this zone, the shrinkage limit decreases as the liquid limit increases. As the liquid limit increases, the rate of change of the shrinkage limit decreases. Based on these results, it is postulated that as the surface area increases, the rate of change of shrinkage limit as the liquid limit increases is progressively decreasing. In Zone 2, which covers montmorillonite clay, the rate of change of shrinkage limit as the liquid limit increases is negligible. It is postulated that within the liquid limit range of 123-228, the surface area of the montmorillonite clay mineral is almost the same; thus, the shrinkage limit of the soil within this range of liquid limit is the same, the reported variation of the shrinkage limit within this range of liquid limit is primarily due to the reproducibility of the test results of ±6.8 as reported in AASHTO T 92-88. in zone 3, for liquid limit of more than 228, the shrinkage limit starts to decrease as the liquid limit increases. It is postulated that at this liquid limit, the surface area of the montmorillonite clay starts to increase, leading to a decrease in the shrinkage limit as the liquid limit increases.

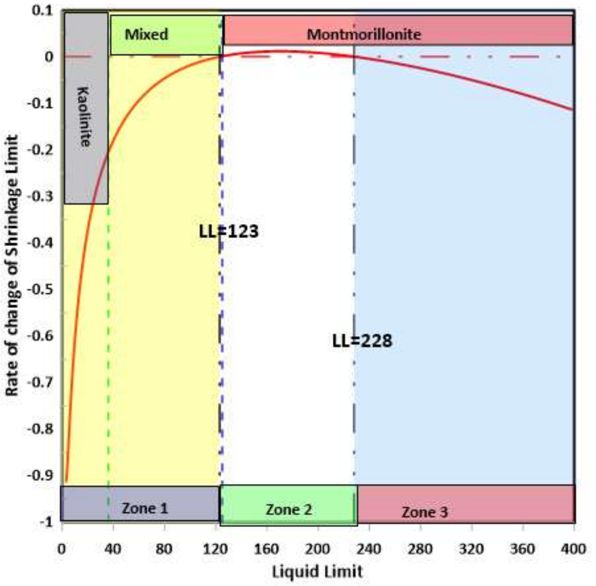

Figure 6: The rate of change of shrinkage limit with liquid limit

Figure 7 shows the position of the liquid limit and shrinkage limit for the test results presented by Srisharan et al. (2000). The bentonite, black cotton soil, brown soils -1 and red earth -1 Atterberg test results were taken directly from Table 1 of the paper while the blended test results were digitized. It can be seen that 8 of 9 test results plot within the statistical testing bound, with only one result marginally plotting outside the statistical testing bound. It is interesting to note also that the trend of shrinkage limit to be almost constant within the liquid limit of 123 to about 228 has been demonstrated by these data.

Based on this validation, it can be concluded that the proposed model of computing the shrinkage limit using the average fall cone slope (equation 4) or using the liquid limit (equation 5) is very robust and can be used to estimate the shrinkage limit of the soil.

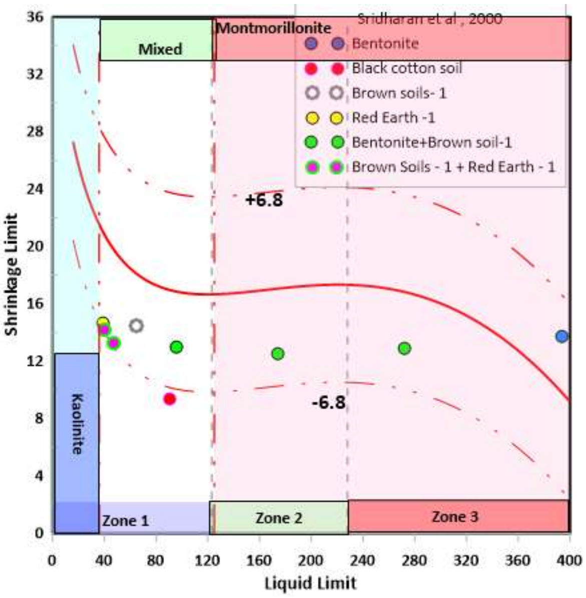

Figure 7: Variation of liquid limit and shrinkage limit

SUMMARY AND CONCLUSION

From the analysis of 181 Atterberg limits test results, the following main conclusion can be drawn from this analysis:

1. The liquid limit is not directly correlated with the shrinkage limit. The lack of correlation is because both liquid and shrinkage limits are boundary values, and they cannot be used to explain the plasticity or shrinkage properties of the soil. However, indirectly, the shrinkage limit is highly correlated to the liquid limit through a correlation between the average fall cone slope (20/LL) and the solidity index (R^2=0.9932), suggesting that the liquid limit can be used to estimate the shrinkage limit of the soil.
2. The plasticity behavior of the soil is best explained by the plasticity index, which is the arithmetic difference between the liquid and plastic limits. Equally, the shrinkage limit cannot be used to describe the shrinkage behavior of the soil. The range of moisture content, such as the solidity index and shrinkage index, can explain the shrinkage properties of soil.
3. The shrinkage limit can be computed using the liquid limit, which is highly correlated to the plasticity index. Therefore, developing a model that computes shrinkage using the plasticity index is also possible. Furthermore, plasticity is imparted to the soil by active clay within the soil matrix. Therefore, the clay content can also be used to compute the shrinkage limit of the soil.
4. Shrinkage limit can be used to delineate different types of clay minerals such as kaolinite, illite, and montmorillonite. Kaolinite has the highest shrinkage limit, while montmorillonite has the lowest value of shrinkage limit.
5. The solidity index is one of the Atterberg limit parameters and is one of the parameters that define the plasticity and shrinkability characteristic of the soil. The solidity index is highly correlated to the average fall cone slope (R^2=0.9932); therefore, the soil's shrinkage limit is also the plasticity characteristic.
6. The specific surface area of the clay minerals influences the solidity index. As the specific surface area increases, the solidity index also increases. Equally, the shrinkage limit is controlled by the specific surface area of the clay minerals; as the specific surface area increases, the shrinkage limit decreases (see Figures 1,5 and 6). Interestingly, the shrinkage limit within the liquid limit range of 123-228 is constant, suggesting that the montmorillonite clay minerals within the range of liquid limit have a constant specific surface area.
7. The conventional method of determining the shrinkage limit of soil involves the use of mercury, which is a hazardous health substance. The equations proposed for computing the shrinkage limit using the

average fall cone slope or liquid limit can be used to compute the shrinkage limit since the results obtained from the proposed equations agree with those from the conventional mercury displacement method and are within acceptable statistical bounds.

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